Closing next wed: HW_2A,2B,2C

Office Hours: 1:30-3:00pm in Com.B-006

Quick review:

Def'n: The "signed" area between $f(x)$ and the x-axis from $x = a$ to $x = b$ is the *definite integral*:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

FTOC(2): If F(x) is <u>any</u> antideriv. of $f(x)$,

$$\int_a^b f(x)dx = F(b) - F(a)$$

Entry Task: Evaluate

$$\int_0^4 e^x + \sqrt{x^3}\,dx \qquad x^{3/2}$$

$$= e^x + \frac{2}{5}x^{5/2}\Big|_0^4$$

$$= \left[e^4 + \frac{2}{5}(4)^{5/2}\right] - \left[e^0 + \frac{2}{5}(0)^{5/2}\right]$$

$$= e^4 + \frac{64}{5} - 1 = \boxed{e^4 + \frac{59}{5}}$$

$$\int_3^6 \frac{4}{x} - \frac{2}{x^2}\,dx = \int_3^6 4\frac{1}{x} - 2x^{-2}\,dx$$

$$= 4\ln|x| - 2\frac{1}{-1}x^{-1}\Big|_3^6$$

$$= 4\ln(x) + \frac{2}{x}\Big|_3^6$$

$$= \left[4\ln(6) + \frac{2}{6}\right] - \left[4\ln(3) + \frac{2}{3}\right]$$

$$= 4\ln(6) + 1/3 - 4\ln(3) - 2/3$$

$$= 4\ln(6) - 4\ln(3) - 1/3$$

$$= \boxed{4\ln(2) - 1/3}$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: The **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$. And we write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is any antiderivative of $f(x)$.

Ex)

$\int x^3 dx = \frac{1}{4}x^4 + C$ = a function

$\int_0^1 x^3 dx = \frac{1}{4}x^4\Big|_0^1 = \frac{1}{4}$ = a number

A brief pause to discuss current integration methods. We can currently find antiderivatives for sums and constant multiples of functions ***directly*** from our integration table.

Examples (we **can** currently do):

1. $\int 6e^x + 4x - 5\sqrt{x}\,dx$

$= 6(e^x) + 4(\frac{1}{2}x^2) - 5(\frac{2}{3}x^{3/2}) + C$

$= \boxed{6e^x + 2x^2 - \frac{10}{3}x^{3/2} + C}$

2. $\int 6\sec^2(x) - \frac{9}{x^4}\,dx$

$\int 6\sec^2(x) - 9x^{-4}\,dx$

$= 6(\tan(x)) - 9(\frac{1}{-3}x^{-3}) + C$

$= \boxed{6\tan(x) + \frac{3}{x^3} + C}$

4.9: LIST OF GENERAL ANTIDERIVATIVES

$\int 11^x dx = \frac{1}{\ln(11)} 11^x + C$

FUNCTION	ANTIDERIVATIVE
$f(x) = x^n \ (n \neq -1)$	$F(x) = \frac{1}{n+1} x^{n+1} + C$
$f(x) = x^{-1} = \frac{1}{x}$	$F(x) = \ln\lvert x\rvert + C$
$f(x) = e^x$	$F(x) = e^x + C$
$f(x) = a^x$	$F(x) = \frac{1}{\ln(a)} a^x + C$
$f(x) = \cos(x)$	$F(x) = \sin(x) + C$
$f(x) = \sec^2(x)$	$F(x) = \tan(x) + C$
$f(x) = \sec(x)\tan(x)$	$F(x) = \sec(x) + C$
$f(x) = \sin(x)$	$F(x) = -\cos(x) + C$
$f(x) = \csc^2(x)$	$F(x) = -\cot(x) + C$
$f(x) = \csc(x)\cot(x)$	$F(x) = -\csc(x) + C$
$f(x) = \frac{1}{1+x^2}$	$F(x) = \tan^{-1}(x) + C$

$e = 2.718281828$

$\frac{d}{dx}(2^x) = 2^x \ln 2$

$\sec(x) = \frac{1}{\cos}$, $\tan(x) = \frac{\sin(x)}{\cos(x)}$

$\csc(x) = \frac{1}{\sin(x)}$, $\cot(x) = \frac{\cos(x)}{\sin}$

$f(x) = a^x \qquad F(x) = \frac{1}{\ln(a)} a^x + C$

Examples we **cannot** currently do (but will be able to do later in the term):

$$\int xe^{3x}dx\,; \qquad \int \tan(x)dx$$

$$\int x\sin(x^2)\,dx; \qquad \int \frac{x^2 - x + 6}{x^3 + 3x}dx$$

$$\int \frac{3}{x - 2\sqrt{x}}dx\,; \qquad \int \frac{\sqrt{x^2 - 1}}{x^2}dx$$

Examples we will *"never"* be able to do:

$$\int e^{x^2}dx\,; \int \sec(x^2)dx$$

Here are two that look bad but we can currently do them, why?

1. $\int \frac{\sqrt{x} - 3x}{x}dx = \int \frac{1}{x}\left(\sqrt{x} - 3x\right)dx$

2. $\int \frac{\cos(x)}{1 - \cos^2(x)}dx$

$\boxed{1}$ $\int x^{-1/2} - 3\,dx$

$= \boxed{2x^{1/2} - 3x + C}$

$\boxed{2}$ $\int \frac{\cos(x)}{\sin^2(x)}dx$

$= \int \frac{\cos(x)}{\sin(x)}\frac{1}{\sin(x)}dx$

$= \int \cot(x)\csc(x)dx$

$= \boxed{-\csc(x) + C}$

What is the value of:

$$\int_{\pi/4}^{\pi/2} \frac{\cos(x)}{1-\cos^2(\mathrm{x})}\,dx$$

$$= -\csc(x)\Big|_{\pi/4}^{\pi/2}$$

$$= \left[-\csc(\pi/2)\right] - \left[-\csc(\pi/4)\right]$$

$$= \left[-\frac{1}{\sin(\pi/2)}\right] - \left[-\frac{1}{\sin(\pi/4)}\right]$$

$$= -1 + \frac{1}{\sqrt{2}/2}$$

$$\boxed{\begin{aligned} &= -1 + \frac{2}{\sqrt{2}} \\ &= -1 + \sqrt{2} \end{aligned}}$$

Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

Let

$s(t)$ = 'location at time t'

$v(t)$ = 'velocity at time t'

pos. $v(t)$ means moving up/right

neg. $v(t)$ means moving down/left

The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

i.e.

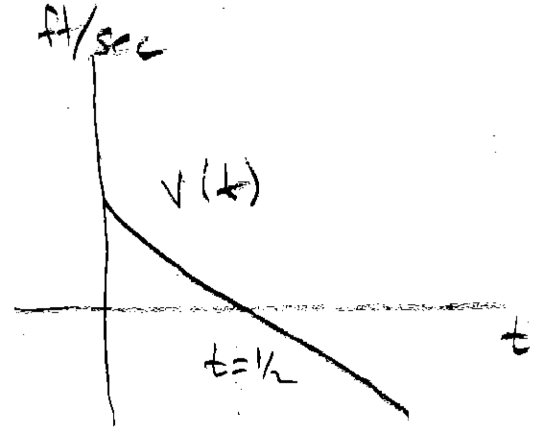
'integral of velocity'= '**net change** in dist'

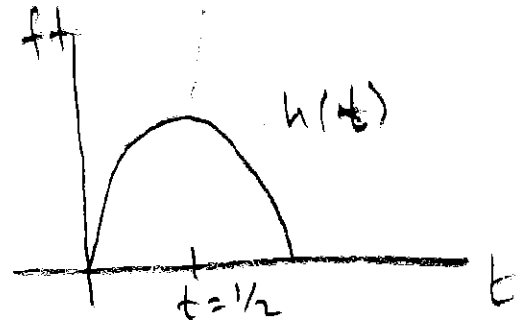
We also call this the *displacement*.

$a(t) = -32$

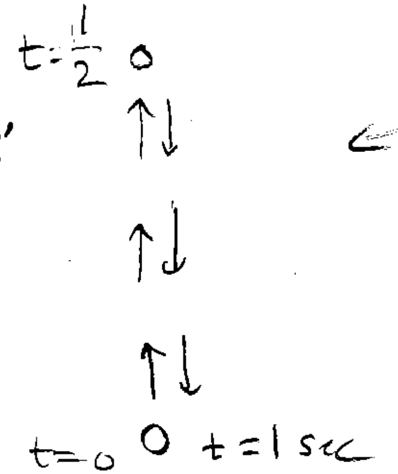
$v(t) = -32t + 16$

$h(t) = -16t^2 + 16t + 0$





ACTUAL VIEW OF OBJECT



$\int_0^1 v(t)dt = 0$

displacement

Thus, in general, the FTOC(2) says the **net change** in $f(x)$ from $x = a$ to $x = b$ is the integral of its **rate**. That is:

$$\int_a^b \underbrace{f'(t)}_{\frac{\text{gal}}{\text{hr}}}\underbrace{dt}_{\text{hr}} = \underbrace{f(b) - f(a)}_{\text{Change in gallons}}$$

$\frac{\text{people}}{\text{yr}}$ yr change in people

etc.

We define **total change** in dist. by

$$\int_a^b |v(t)|dt$$

which we compute by

1. Solving $v(t) = 0$ for t.
2. Splitting up the integral at these t values, dropping the absolute value and integrating separately.
3. Adding together as positive numbers.

Example: $v(t) = t^2 - 2t - 8$ ft/sec
Compute the total distance traveled from t = 1 to t = 6.

$$\int_1^6 |t^2 - 2t - 8|\,dt$$

$\boxed{1}$ $t^2 - 2t - 8 \overset{?}{=} 0$

$(t-4)(t+2) = 0$

$t = 4$ and $t = -2$

$\boxed{2}$ $\int_1^4 t^2 - 2t - 8\,dt = -18$ ft

$\int_4^6 t^2 - 2t - 8\,dt = \frac{44}{3} = 14.\overline{6}$ ft

The object goes "left" 18 feet then "right" $14.\overline{6}$ feet

For $\boxed{\text{TOTAL DISTANCE} = 18 + 14.\overline{6} = 32.\overline{6} \text{ ft}}$

NOTE: DISPLACEMENT $= \int_1^6 t^2 - 2t - 8\,dt = -18 + 14.\overline{6}$
$= -3.\overline{3}$ ft